The Entropic Potential of an Event in Deterministic and Indeterministic Systems

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Abstract

This article analyses entropy changes triggered by specific events in deterministic and indeterministic systems. The article considers a simple model consisting of water in a cuvette, an ice cube in the device above the cuvette and a random number generator (RNG) that controls the probability of dropping the ice into water.

The article introduces the entropic potential Z(T, A) of an event A occurred in a system R at the moment T_0, which describes the influence of the event A to the entropy of the system R in the future (at $T>T_0$). The entropic potential of an event Z(T, A) can be calculated as the difference between the mathematical expectations of entropy of the system R for the moment T ($T>T_0$) made immediately before and immediately after the event A as $Z(T, A) = \hat{S}_T(T_0 + dT) – \hat{S}_T(T_0 – dT)$. The article also presents examples of calculations of the entropic potentials of events in indeterministic systems with different probabilities of events.

Since real-life systems are mostly indeterministic, the entropic potentials of events in real-life usually have non-zero values. The entropic potentials of the events "useful" for the system are negative, and entropic potentials of the events "harmful" for the system are positive.

Keywords: entropy; event; deterministic system; indeterministic system; entropic potential of an event

1. Introduction. An analysis of entropy changes in deterministic and indeterministic systems

1.1. Deterministic system

Let us consider a simple system consisting of water in a cuvette and an ice cube located in the device above this cuvette. Let the moment T_0 designate when the device drops the ice into water. The ice melts and during the specific moment, T_{fin} , the system enters a state of equilibrium. Since in the described model the system's entropy S is a function of time T, let us denote it as S(T). At time T_0 it is $S(T_0)$ and at the moment T_{fin} it is $S(T_{fin})$. During the ice melting the entropy grows and we denote this increase as ΔS.

$$\Delta S = S(T_{fin}) - S(T_0) \qquad (1)$$

We can also calculate the average speed of entropy growth (V_{eg}), which is

$$V_{eg} = [S(T_{fin}) - S(T_0)] / (T_{fin} - T_0) \qquad (2)$$

For an arbitrary moment of time (T) between the immersion of ice and the equilibrium state ($T_0<T<T_{fin}$) the entropy of the system is between $S(T_0)$ and $S(T_{fin})$ and for the time (T) after reaching the equilibrium ($T>T_{fin}$), the entropy of the system stays as $S(T_{fin})$ while the system is isolated. (For simplicity we neglect the melting of ice while it is located in the device before it drops into the water.)

1.2. Indeterministic systems

The model above with the ice cube dropping into water represents a deterministic system [1], [2]. In that model, the probability of the piece of ice dropping into water is 100%.

Let us now consider an *indeterministic* system [3] where the probability of dropping the ice into water is less than 100%. It can, for example, be a system where the device drops the piece of ice into water *only* if a random number generator (RNG) [4] connected to this device returns a specific predefined number. For simplicity we check the random number generator only *once.* This restriction is important to avoid the situation when the ice will *eventually* be dropped into water after multiple RNG trials. Let us denote the moment of the RNG trial as T_0 and the probability that the RNG outcome will trigger the ice drop as p. In the indeterministic system at the time preceding the RNG trial, we do not know for sure if the ice will be dropped into the water or not, and correspondingly we have to use the mathematical expectation $\hat{S}(T_{fin})$ as an estimation of system entropy at the moment T_{fin} instead of the $S(T_{fin})$ value.

To analyze this model in further detail we will consider two moments of time. One moment is (T_0-dT), which *immediately precedes* the RNG trial. The second moment is (T_0+dT), which *immediately follows* the RNG trial. Similarly to the deterministic model, T_{fin} denotes the moment in the future in which the system would achieve equilibrium if the ice was dropped at the time T_0. Since the temperature of the water and ice and their masses are known in this model, the time T_{fin} is calculatable.

Let us try to estimate the system entropy S_{fin} for the moment T_{fin} in the future. We will perform two estimations at the moments (T_0-dT) and (T_0+dT) and will also denote these entropy estimations as $\hat{S}_{fin}(T_0-dT)$ and $\hat{S}_{fin}(T_0+dT)$.

When we estimate entropy S_{fin} at the moment T_0-dT we do not yet know if the ice will be dropped into the water or not. Correspondingly, the mathematical expectation of S_{fin} is

$$\hat{S}_{fin}(T_0-dT) = S(T_0) + \Delta S*p \qquad (3),$$

where the change in entropy ΔS was defined in formula (1) and p is the probability that the RNG outcome will trigger the ice dropping.

When we estimate entropy $\hat{S}_{fin}$ at the moment T_0+dT we already know if the ice was dropped into the water or not. Here we can have one of two cases.

If the RNG produced the number that triggered the ice drop, then the estimation of entropy $\hat{S}_{fin}$ is

$$\hat{S}_{fin}(T_0+dT)= S(T_0) + \Delta S \qquad (4).$$

If the RNG did not produce the number that triggered the ice drop, then the estimation of entropy $\hat{S}_{fin}$ is

$$\hat{S}_{fin}(T_0+dT)= S(T_0) \qquad (5),$$

since the initial state was not changed and entropy did not increase.

1.3. Calculation of the strength of the "RNG trial" event to the change of entropy in the future

As we saw in the presented model, a random event in the system at the moment T_0 may influence the system entropy value at the moment of T_{fin} in the future. The above formulae also give us the method to calculate the strength of this influence. Before the RNG trial the estimation of entropy for the moment T_{fin} was $\hat{S}_{fin}(T_0$-dT). After the RNG trial the estimation of entropy for the moment T_{fin} is $\hat{S}_{fin}(T_0$+dT). Correspondingly we can use the difference of the math estimations $\hat{S}_{fin}(T_0+dT) - \hat{S}_{fin}(T_0-dT)$ to measure the influence of the RNG outcome to entropy changes in the future.

For estimating the influence of the event "*RNG outcome has triggered the ice drop*" to the system entropy in the future we have the difference

$$\hat{S}_{fin}(T_0+dT) - \hat{S}_{fin}(T_0-dT) = [S(T_0) + \Delta S] – [S(T_0) + \Delta S*p] = \Delta S(1-p) \qquad (6).$$

For estimating the influence of the event "*RNG outcome did NOT trigger the ice drop*" to the system entropy in the future we have the difference

$$\hat{S}_{fin}(T_0+dT) - \hat{S}_{fin}(T_0-dT) = S(T_0) - [S(T_0) + \Delta S*p] = - \Delta S*p \qquad (7).$$

This way we can describe the strength of influence of random events occurred in the system to the speed of future entropy growth in this system.

2. Entropic potential of an event

The text above illustrated the preamble of the introduction of an *Entropic potential of an event*, which describes the influence of an event occurred at the moment T_0 to the entropy change in the future $T>T_0$.

2.1. Definition

The entropic potential Z (T, A) of an event A occurred in the system R at the moment T_0 *describes the influence of the event A to the entropy of the system R in the future (for the moments* $T>T_0$*).*

As shown in the above model,

a. in indeterministic systems the entropy can grow faster or slower depending on the random events occurred in this system,
b. the strength of influence of these events to the speed of entropy growth can be formulated.

2.2. Formula

As shown in the section "Indeterministic systems", the entropic potential Z(T, A) can be formulated as the difference between the mathematical expectations of entropy of the system R for the moment T ($T>T_0$) made immediately before and immediately after the event A.

$$Z(T, A) = \hat{S}_T(T_0 + dT) - \hat{S}_T(T_0 - dT) \qquad (8)$$

2.3. Examples of "Entropic potential of an event" calculations

Let us return to the formulae (6) and (7) and review in further detail what they mean from a physical point of view. As above, ΔS is the entropy increase, which occurs if the ice is dropped into water. The value p shows the probability of this event. Since p is a probability, it is located within the [0,1] interval. Let us analyze several cases.

a. *The case when the system is deterministic.* In such a case the probability is p=1 because the device drops the ice into water for any RNG outcome. Correspondingly the mathematical expectations made before and after the RNG trial are the same, $\hat{S}_{fin}(T_0+dT) = \hat{S}_{fin}(T_0-dT)$. Correspondingly, the entropic potential of the RNG trial is zero Z(A,T)=0, because the outcome of the RNG at the moment T_0 does not influence the change of entropy in the future.

b. *The case when the system is indeterministic and probability p is high, p~1.*

   For example, this can be implemented if from the n available RNG outcomes the (n-1) outcomes lead to dropping the ice into water. The entropic potential of the event "*RNG trial has triggered the ice drop*" is described by formula (6)

$$Z(A,T_{fin}) = \Delta S(1\text{-}p) \approx 0$$

   The entropic potential of the "*RNG trial has triggered the ice drop*" event is small because it is an expected event with high probability p=(n-1)/n. It occurs in (n-1) cases from n and correspondingly the event "*RNG trial has triggered the ice drop*" practically does not alter the entropy at the moment T_{fin} from its mathematical expectation made before the RNG trial.

   The entropic potential of the event "*RNG trial did NOT trigger the ice drop*" is described by formula (7)

$$Z(A,T_{fin}) = -\Delta S{*}p \approx -\Delta S$$

   The potential $Z(A,T_{fin})$ in this case is high and has a negative value because the event "*RNG trial did NOT trigger the ice drop*" has a low probability. Reasonably it is expected that the entropy at the moment T_{fin} will be $\hat{S}_{fin}=S_0 + \Delta S$. However, because of the RNG event "*RNG trial did NOT trigger the ice drop*", the device did not drop the ice into water and entropy S_{fin} at the moment T_{fin} will be equal to $S_{fin} = S_0$, which is much less than its mathematical expectation $S(T_0) + \Delta S{*}p \approx S(T_0) + \Delta S$. The entropy growth halts and correspondingly the event "*RNG trial did NOT trigger the ice drop*" has a large negative entropic potential. This slows down the growth of entropy (in this model it even halts it) in the system R compared to the expected outcome.

c. *The case when the system is indeterministic and probability p is 0.5.*

For example, this can be implemented if within the available n RNG trials only the n/2 outcomes lead to the dropping of ice into water. The entropic potential of the event "*RNG trial has triggered the ice drop*" is described by formula (6), which is shown as

$$Z(A,T_{fin}) = \Delta S(1\text{-}p) = \Delta S/2$$

The entropic potential of the event "*RNG trial did NOT trigger the ice drop*" is described by formula (7), which now is shown as

$$Z(A,T_{fin}) = -\Delta S(1\text{-}p) = -\Delta S/2$$

As shown, the absolute values of entropic potentials for both events are equal. However, the potentials have different signs. The following shows why.

The event "*RNG trial has triggered the ice drop*" causes the entropy S_{fin} to be higher than the mathematical expectation. The estimation $\hat{S}_{fin}(T_0\text{-}dT)$ made before the RNG trial produces the value $S(T_0) + \Delta S{*}p= S(T_0) + \Delta S/2$. However, the event "*RNG trial has triggered the ice drop*" produces the entropy $S_{fin}= S(T_0) + \Delta S$ at the moment T_{fin}, which is $\Delta S/2$ higher than the mathematical expectation made before the RNG trial. Correspondingly, the entropic potential of this event is positive, and the system's entropy grows faster than estimated.

In the inverse case, the event "*RNG trial did NOT trigger the ice drop*" leads to the entropy $S_{fin}= S(T_0)$ at the moment T_{fin}, which is $\Delta S/2$ lower than the mathematical expectation made before the RNG trial. Correspondingly, the entropic potential of this event is negative and the system's entropy grows slower than estimated.

d. *The case when the system is indeterministic and probability p is low, p~0.*

This can be implemented if within the n available RNG results only 1 outcome leads to the dropping of ice into water. The entropic potential of the event "*RNG trial triggers the ice drop*" is described by formula (6)

$$Z(A,T_{fin}) = \Delta S(1\text{-}p) \approx \Delta S$$

As shown, the entropic potential of the "*RNG trial has triggered the ice drop*" event is high because it is an unexpected event with low probability p=1/n. It occurs only in 1 case from n and correspondingly the event "*RNG trial triggered the ice drop*" significantly influences the entropy of the system R at the moment T_{fin} deviating it from the mathematical expectation. In this case the mathematical expectation is $\hat{S}_{fin}(T_0\text{-}dT) = S(T_0) + \Delta S{*}p \approx S(T_0)$, while S_{fin} is $S_{fin} = S(T_0) + \Delta S \gg S(T_0)$.

The entropic potential of the event "*RNG trial did NOT trigger the ice drop*" is described by formula (7)

$$Z(A,T_{fin}) = - \Delta S{*}p \approx 0.$$

The potential $Z(A,T_{fin})$ in this case is negative and very small. The following shows why. In the analyzed case, the event "*RNG trial did NOT trigger the ice drop*" has a high probability. All the RNG outcomes but one keeps the ice out of water. Therefore, the event "*RNG trial did NOT trigger the ice drop*" practically does not deviate the final entropy $S_{fin} = S(T_0)$ from its mathematical expectation $\hat{S}_{fin}(T_0\text{-}dT) = S(T_0) + \Delta S{*}p \approx S(T_0)$. Correspondingly, the event "*RNG trial did NOT trigger the ice drop*" at T_0 practically does not influence the entropy at the future moment T_{fin}.

3. Discussion. The Entropic Potential of an Event and real-life systems

Real-life systems are mostly indeterministic as for almost any event A the event Ā ("not A"), with the non-zero probability $p(\bar{A}) = 1\text{-}p(A) \neq 0$ can also occur. Correspondingly, the entropic potentials of real-life events are not zero, $Z(A,T) \neq 0$ for any T as soon as the entropy in these systems is not constant.

Since system entropy describes the amount of disorder/organization in the system, the entropic potential of events Z(A,T) thereby describes the impact of event A to the development or degradation of the system R in the future. Therefore, if before event A has occurred (i.e., when it is not yet known if A or Ā will take place) a mathematical expectation of entropy at the moment T is $\hat{S}\,(T_0\text{-}dT)$ and after event A has occurred the mathematical expectation of entropy is $\hat{S}_T\,(T_0\text{+}dT)$ and $\hat{S}_T\,(T_0\text{+}dT) < \hat{S}_T\,(T_0\text{-}dT)$. This in essence means that event A prevented the growth of entropy in the system R, and protected it from degradation and destruction at least until the moment T. For such cases the entropic potential of event A is negative $Z(T, A) = \hat{S}_T(T_0 + dT) – \hat{S}_T(T_0 – dT) < 0$.

Vice versa, if the difference between the mathematical expectations of entropy at the future moment T is calculated after and before the event A is positive $\hat{S}_T(T_0+dT) > \hat{S}_T(T_0-dT)$, this essentially means that the event A sped up the growth of entropy in the system R and accelerated degradation of this system at least until the moment T. For such cases the entropic potential of event A is positive $Z(T, A) = \hat{S}_T(T_0 + dT) - \hat{S}_T(T_0 - dT) > 0$.

In short, *the entropic potentials of the events "useful" for system R are negative, and the entropic potentials of the events "harmful" for system R are positive*.

References

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